INTRO TO EQUATIONS

❑ INTRODUCTION

I'm thinking of a number. If I add 10 to the number, the result is 45. What number was I thinking of ?

Remember this question from the chapter *Constants*? Now we're ready to formalize the thought processes you went through to solve this problem. It may seem silly to make a big deal about this easy problem, but the skills you develop now will give you the tools needed to solve problems that could never be solved in your head.

An equation must always be in balance.

❑ ONE-STEP EQUATIONS

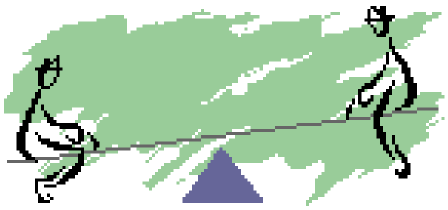
Considering the problem again, let's pretend we don't already know the answer. We'll choose some symbol to represent the unknown number; call it n. Here's what we need to do: Translate the English statement

"*If I add* 10 *to the number, the result is* 45"

into an equation, a statement of equality. Since we let n represent the number, then the phrase "*If I add 10 to the number*" becomes "$n + 10$", the phrase "*the result is*" becomes " = ", and the 45 is, of course, just 45. The final translation from English to algebra results in the equation

$$\underbrace{n+10}_{\text{If I add 10 to the number}} \quad \underbrace{=}_{\text{the result is}} \quad \underbrace{45}_{45}$$

An equation is like a seesaw in balance, perfectly level with a child at each end. If we were to drop a bowling ball onto the lap of one child, that child would plummet to the ground, throwing the second child into the air — all in all, not a pleasant situation. But most importantly, the seesaw would not be in balance anymore; one end would be on the ground and the other up in the air. But what if we dropped a 10-lb



bowling ball on the lap of each child at exactly the same moment? Even with kids screaming in pain, at least the seesaw would remain level, still in balance. By the way, math teachers don't care about kids in pain, as long as the equation remains balanced!

This is the essence of solving an equation:

As long as we perform the <u>same</u> operation
on <u>each</u> side of an equation,
the resulting statement should
still be a balanced equation.

Thus, to solve the equation above, $n + 10 = 45$, we need to do something to each side of the equation that will tell us what n is. We have to "isolate" the n. To do that, we have to strip away the 10 from the expression $n + 10$.

Here's the thought process: The operation connecting the n with the 10 is addition. The operation which removes addition is subtraction. Therefore, we need to subtract 10 from the left side of the equation, which will leave the n all by itself (isolated). *But whatever we do to one side of an equation, we must do to the other side.*

$n + 10 = 45$ (the original equation)

$\Rightarrow \quad n + 10 - \mathbf{10} = 45 - \mathbf{10}$ (subtract 10 from each side of the equation)

$\Rightarrow \quad n = 35$ (simplify each side)

Therefore, the number I was thinking of was $\boxed{35}$

The key to solving any equation:

DO THE **SAME** THING TO EACH SIDE OF THE EQUATION.

<u>EXAMPLE 1:</u> **Solve each equation:**

A. $x + 17 = 50$

Since the x and the 17 are connected by addition, we will use subtraction to remove the 17. Remembering to always do the same thing to each side of the equation, we get

$x + 17 - \mathbf{17} = 50 - \mathbf{17}$ (subtract 17 from each side)

$\boldsymbol{x} = \mathbf{33}$ (simplify each side)

B. $y - 12 = 13$

In this equation, the 12 is being subtracted from the unknown y; to undo the subtracting, we add 12 to each side of the equation:

$y - 12 \mathbf{+ 12} = 13 \mathbf{+ 12}$ (add 12 to each side)

$\boldsymbol{y} = \mathbf{25}$ (simplify each side)

C. $7n = 35$

This time the operation to remove is multiplication, which is easily removed by division:

$\dfrac{7n}{\mathbf{7}} = \dfrac{35}{\mathbf{7}}$ (divide each side by 7)

$\boldsymbol{n} = \mathbf{5}$ (simplify each side)

D. $\frac{a}{8} = 2$

The operation connecting the unknown a with the 8 is division. Since the reverse of division is multiplication, we shall multiply each side of the equation by 8:

$\left(\frac{a}{8}\right)\mathbf{8} = (2)\mathbf{8}$ (multiply each side by 8)

$\left(\frac{a}{\not 8}\right)\not 8 = 16$ (cross-cancel the 8's)

$\boldsymbol{a} = \mathbf{16}$ (simplify)

This is what happens if you <u>don't</u> do the same thing to each side of the equation.

Homework

1. Solve each equation:

a. $w + 17 = 25$ b. $a - 9 = 4$ c. $x - 2 = 90$

d. $y + 99 = 105$ e. $z - 32 = 80$ f. $b + 50 = 160$

g. $8 + x = 12$ h. $12 + t = 100$ i. $30 + m = 30$

j. $n - 17 = 50$ k. $J + 20 = 44.9$ l. $3.3 + T = 100$

m. $w - 4.5 = 8.2$ n. $u + 4 = 1000$ o. $n + 0.88 = 0.88$

2. Solve each equation:

a. $3x = 18$ b. $2n = 144$ c. $5u = 95$

d. $4z = 30.4$ e. $1.3m = 128.7$ f. $0.25Q = 0.45$

g. $7x = 28$ h. $32a = 128$ i. $5R = 1000$

j. $1000k = 10{,}000$ k. $2x = 17$ l. $7h = 14$

m. $12c = 12$ n. $18a = 0$ o. $2x = 10$

3. Solve each equation:

a. $\frac{x}{3} = 6$ b. $\frac{y}{4} = 24$ c. $\frac{z}{2} = 33$

d. $\frac{a}{23} = 5$ e. $\frac{b}{17} = 34$ f. $\frac{c}{88} = 1$

g. $\frac{x}{7} = 1$ h. $\frac{a}{30} = 30$ i. $\frac{u}{17} = 3$

j. $\frac{y}{0.5} = 20$ k. $\frac{p}{1.7} = 0.4$ l. $\frac{q}{2.3} = 2.3$

4. Solve each equation:

a. $2x = 18$ b. $3y = 16$ c. $4n = 3$

d. $7 + x = 9$ e. $x - 3 = 42$ f. $y + 9.3 = 9.3$

g. $n(7) = 49$ h. $z - 1 = 1$ i. $a + 2.3 = 9.1$

j. $b - 0.7 = 2$ k. $c + 3 = 4.3$ l. $d - 2.13 = 18.77$

m. $\frac{x}{3} = 7$ n. $\frac{y}{9} = 1$ o. $\frac{z}{1.3} = 0.5$

❑ *TWO-STEP EQUATIONS*

Here's another word problem that can be done by guessing or working backwards:

> *"I'm thinking of a number. If I multiply it by 3 and then add 4, the result is 34. What is the number I was thinking of?"*

Translating this problem into algebra (assuming n is the unknown number) results in the equation

$$3n + 4 = 34$$

As we learned above, since the 3 is connected to the n by multiplying, we rid ourselves of the 3 by dividing. And because the 4 is being added to the $3n$, we remove the 4 by subtracting. And, of course, whenever we do something to the left side of the equation, we'll keep things in balance by doing the same thing to the right side.

This leaves one important question:

Which do we get rid of first, the 3 or the 4?

Just like unwrapping a gift, we start with the final operation in the expression, and work backwards until the variable n is alone. The final operation in the expression $3n + 4$ is addition, so we get rid of the 4 first:

$$3n + 4 = 34 \quad \text{(the original equation)}$$

$$\Rightarrow \quad 3n + 4 - \mathbf{4} = 34 - \mathbf{4} \quad \text{(subtract 4 from each side)}$$

$$\Rightarrow \quad 3n = 30 \quad \text{(simplify each side)}$$

$$\Rightarrow \quad \frac{3n}{\mathbf{3}} = \frac{30}{\mathbf{3}} \quad \text{(divide each side by 3)}$$

$$\Rightarrow \quad n = 10 \quad \text{(simplify each side)}$$

and we've found that the unknown number is $\boxed{10}$

Note: Since the expression $3n + 4$ was created using the Order of Operations – multiply, then add – we undo the order of operations by going in the reverse order: First we undo the last operation, addition, and second we undo the first operation, multiplication.

The next example shows us how we generally leave fractional answers in algebra.

EXAMPLE 2: **Solve for x: $-14 + 8x = 22$**

Solution:

Add 14 to each side of the equation:	$-14 \mathbf{+ 14} + 8x = 22 \mathbf{+ 14}$
Simplify:	$8x = 36$
Divide each side by 8:	$\frac{8x}{\mathbf{8}} = \frac{36}{\mathbf{8}}$
Simplify:	$\boxed{x = \frac{9}{2}}$

Notice that it's okay in algebra to leave an improper fraction as your final answer, as long as it's in reduced form. In this example, the answer $\frac{36}{8}$ must be reduced to $\frac{9}{2}$, but it need not be converted to a mixed number.

Some students find it easier by first using the commutative property of addition to flip the terms on the left side of the given equation:

$$8x - 14 = 22$$

EXAMPLE 3: **Solve for a: $\frac{a}{4} - 2 = 10$**

Solution: We notice that in this equation, the first operation on the variable a is the division, followed by the subtraction. Therefore, our plan is to reverse these steps.

The equation we're to solve:	$\frac{a}{4} - 2 = 10$
Add 2 to each side of the equation:	$\frac{a}{4} - 2 \mathbf{+ 2} = 10 \mathbf{+ 2}$
And simplify:	$\frac{a}{4} = 12$
Last, multiply each side of the equation by 4:	$\frac{a}{4}[\mathbf{4}] = 12[\mathbf{4}]$
And we're done:	$\boxed{a = 48}$

<u>EXAMPLE 4:</u> **Solve for *n*:** $\mathbf{\frac{n+7}{3} = 9}$

<u>Solution:</u> In this equation the final operation is division, so our first step in solving the equation is to multiply.

$$\frac{n+7}{3}[\mathbf{3}] = 9[\mathbf{3}] \qquad \text{(multiply each side by 3)}$$

$$\Rightarrow \quad n + 7 = 27 \qquad \text{(simplify each side)}$$

$$\Rightarrow \quad n + 7 \mathbf{- 7} = 27 \mathbf{- 7} \qquad \text{(subtract 7 from each side)}$$

$$\Rightarrow \quad \boxed{n = 20} \qquad \text{(simplify each side)}$$

Homework

5. Solve each equation, leaving answers as fractions:

a. $3n + 7 = 22$ b. $2x - 1 = 49$ c. $4q + 17 = 93$

d. $7z - 7 = 7$ e. $13w + 26 = 65$ f. $23x + 18 = 38$

g. $18u - 7 = 11$ h. $12v + 80 = 80$ i. $7a + 3 = 10$

j. $n + 17 = 17$ k. $23T - 44 = 55$ l. $12 + 5k = 20$

m. $7x + 12 = 100$ n. $3y - 13 = 20$ o. $7t + 18 = 18$

p. $13c - 98 = 1$ q. $23 + 7h = 24$ r. $12 + 5w = 15$

6. Solve each equation:

a. $\frac{x+4}{5} = 2$ b. $\frac{a}{3} - 4 = 5$ c. $\frac{u-1}{3} = 3$

d. $\frac{c-3}{4} = 0$ e. $\frac{y}{7} + 1 = 5$ f. $\frac{w}{2} - 7 = 5$

g. $\frac{n+3}{5} = 3$ h. $\frac{b}{4} - 3 = 1$ i. $\frac{z-1}{4} = 4$

j. $\frac{d-7}{\pi} = 0$ k. $\frac{m}{6} + 2 = 5$ l. $\frac{g}{2} - 70 = 500$

Solutions

1. a. $w = 8$ b. $a = 13$ c. $x = 92$ d. $y = 6$
e. $z = 112$ f. $b = 110$ g. $x = 4$ h. $t = 88$
i. $m = 0$ j. $n = 67$ k. $J = 24.9$ l. $T = 96.7$
m. $w = 12.7$ n. $u = 996$ o. $n = 0$

2. a. $x = 6$ b. $n = 72$ c. $u = 19$ d. $z = 7.6$
e. $m = 99$ f. $Q = 1.8$ g. $x = 4$ h. $a = 4$
i. $R = 200$ j. $k = 10$ k. $x = 8.5$ l. $h = 2$
m. $c = 1$ n. $a = 0$ o. $x = 5$

3. a. $x = 18$ b. $y = 96$ c. $z = 66$ d. $a = 115$
e. $b = 578$ f. $c = 88$ g. $x = 7$ h. $a = 900$
i. $u = 51$ j. $y = 10$ k. $p = 0.68$ l. $q = 5.29$

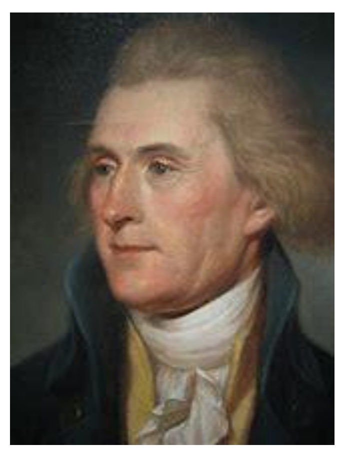
4. a. $x = 9$ b. $y = 5.33$ c. $n = 0.75$ d. $x = 2$
e. $x = 45$ f. $y = 0$ g. $n = 7$ h. $z = 2$
i. $a = 6.8$ j. $b = 2.7$ k. $c = 1.3$ l. $d = 20.9$
m. $x = 21$ n. $y = 9$ o. $z = 0.65$

5. For this problem, answers will be written in fraction form, just for variety. Notice that it's okay in algebra to leave an improper fraction as your final answer, as long as it's been reduced. For example, the answer $\frac{10}{8}$ must be reduced to $\frac{5}{4}$, but it need not be converted to a mixed number.

a. $n = 5$ b. $x = 25$ c. $q = 19$ d. $z = 2$
e. $w = 3$ f. $x = \frac{20}{23}$ g. $u = 1$ h. $v = 0$
i. $a = 1$ j. $n = 0$ k. $T = \frac{99}{23}$ l. $k = \frac{8}{5}$
m. $x = \frac{88}{7}$ n. $y = 11$ o. $t = 0$ p. $c = \frac{99}{13}$
q. $h = \frac{1}{7}$ r. $w = \frac{3}{5}$

6. a. $x = 6$ b. $a = 27$ c. $u = 10$
d. $c = 3$ e. $y = 28$ f. $w = 24$
g. $n = 12$ h. $b = 16$ i. $z = 17$
j. $d = 7$ k. $m = 18$ l. $g = 1,140$

"If a nation expects to be ignorant and free, it expects what never was, and will never be."



– Thomas Jefferson